



TETHER TRIGONOMETRY | EDUCATOR

Links to Common Core Standards |

CCSS.MATH.CONTENT. 6.G.A.4:

Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems.

CCSS.MATH.CONTENT. 8.G.B.7:

Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.

CCSS.MATH.CONTENT. 8.G.C.9:

Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems.

STEM

Pacing | One class period or less

Background Needed | Introduction to right triangle trigonometry, Pythagorean Theorem, and the formula for determining area of a circle

Assessment | An answer key is provided for worksheet problems

Materials/Resources |

- ▶ Tether Trigonometry student worksheets

Overview

Students will gain a practical application for right triangle trigonometry in solving for vertical and linear distance relationships between remotely operated vehicles underwater. Students will build on this mathematic relationship to find the operation area of vehicles using the formula for area of a circle. Worksheet problems challenge students to use multiple equation strategies to solve these underwater problems.

Objectives & Learning Outcomes

- ▶ Students will be able to determine unknown side lengths in right triangles.
- ▶ Students will be able to determine the area of a circle by solving for the radius using real-world problems and trigonometry.

Guiding Questions

- ▶ What are some real-world applications for trigonometry?
- ▶ How can the ROVs position themselves to provide the largest amount of reachable seafloor to explore?

Important Equations |

Area of a circle: $A = \pi r^2$

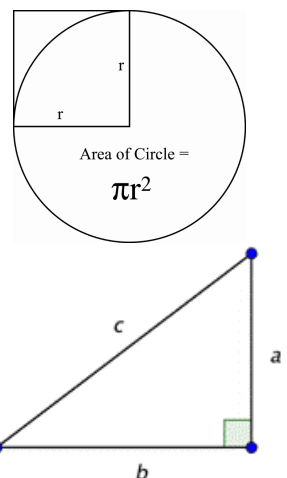
A = area

r = radius

Right angle trigonometry:

In a right triangle, if you know the length of any two sides you can use the Pythagorean Theorem to find the length of the third side.

Pythagorean Theorem: $a^2 + b^2 = c^2$





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Set The Stage! Use the following video, also highlighted in the student module, to provide a brief background on the operational system and teamwork involved with using Remote Operated Vehicles *Hercules* and *Argus*: nautl.us/1VfIKWx.

Vocabulary I

The following terms are used throughout the worksheet. Depending on your class needs, you may want to review some of this terminology with the group before assigning the worksheet problems.

Depth

- ✓ A distance measured from the ocean surface to a specific target; in this case the depth refers to the distance from the surface to each remotely operated vehicle.

Delta depth

- ✓ The difference (delta) in depth between the two remotely operated vehicles.

ROV *Hercules*

- ✓ The highly maneuverable remotely operated vehicle with manipulator arms for collecting samples and a variety of scientific equipment. The vehicle sits below ROV *Argus* and delivers high resolution video and streaming data to scientists and viewers globally.

ROV *Argus*

- ✓ The remotely operated vehicle attached to an inflexible, heavy armored steel cable directly back to E/V *Nautilus*. *Argus* hangs straight down beneath the ship acting as a shock absorber for *Hercules* by reducing the amount of heave (up and down motion) felt by the ship. *Argus* also provides bright lights to illuminate the seafloor for ROV *Hercules*.



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Extensions & Adaptations

Introductory I

Help students visualize the arrangement of the two ROVs by using string and pendulums, washers, or beads. In place of calculations, students can measure the distances between the two vehicles or sketch the different layouts with graph paper and count the distances. Select triangle side distances that fit a 3:4:5 ratio for easy computation.

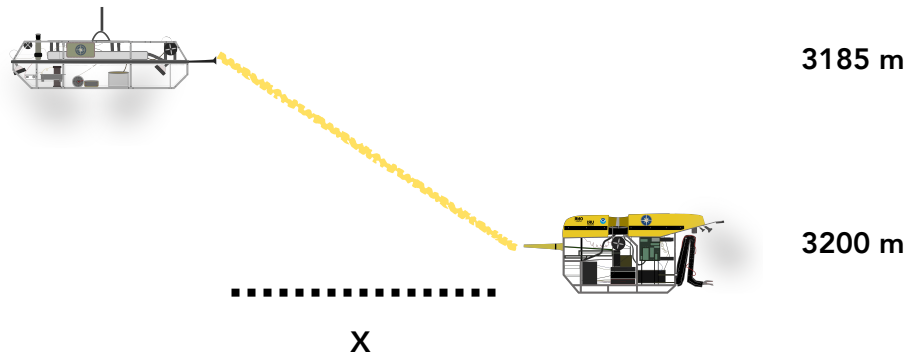
Advanced I

Challenge students to calculate the total volume of exploration space available to the ROVs within a certain configuration. The solution would be a column- shape of a set diameter underneath ROV Argus.

Assuming the seafloor is flat: Challenge students to find the optimal delta depth to provide ROV Hercules with the largest surface area of seafloor to explore.

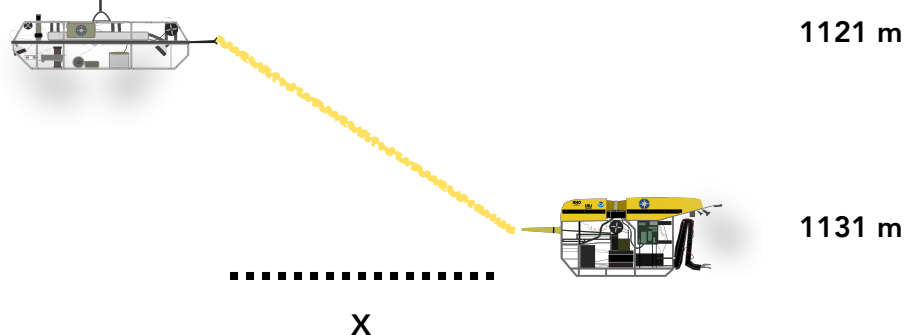
Section One - Tether Management

1. If Argus and Hercules are connected with a 30 meter tether, solve for the horizontal range indicated by x.



$$\begin{aligned}
 a^2 + b^2 &= c^2 & \text{OR } \text{delta depth}^2 + x^2 &= \text{tether}^2 \\
 (3200-3185)^2 + x^2 &= 30^2 \\
 225 + x^2 &= 900 \\
 x^2 &= 675 \\
 x &= \sqrt{675} \\
 \mathbf{x} &= \mathbf{26 \text{ meters}}
 \end{aligned}$$

2. If Argus and Hercules are connected with a 50 meter tether, solve for the horizontal range indicated by x.



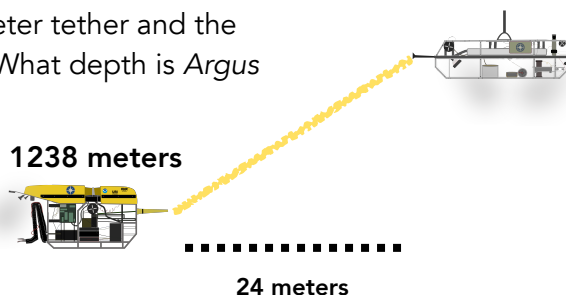
$$\begin{aligned}
 a^2 + b^2 &= c^2 & \text{OR } \text{delta depth}^2 + x^2 &= \text{tether}^2 \\
 (1131-1121)^2 + x^2 &= 50^2 \\
 100 + x^2 &= 2500 \\
 x^2 &= 2400 \\
 x &= \sqrt{2400} \\
 \mathbf{x} &= \mathbf{49 \text{ meters}}
 \end{aligned}$$

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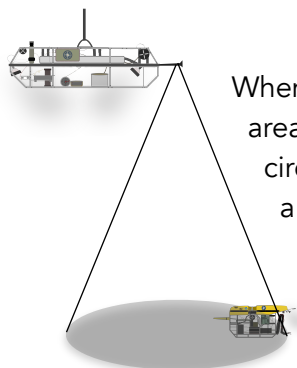


3. *Argus* and *Hercules* are connected with a 30 meter tether and the horizontal range is 24 meters across the seafloor. What depth is *Argus* sitting at?

$$\begin{aligned}
 a^2 + b^2 &= c^2 \\
 \text{horizontal range}^2 + \text{delta depth}^2 &= \text{tether}^2 \\
 24^2 + \text{delta depth}^2 &= 30^2 \\
 576 + x^2 &= 900 \\
 \text{delta depth}^2 &= 324 \\
 \text{delta depth} &= \sqrt{324} \\
 \text{delta depth} &= 18 \text{ meters}
 \end{aligned}$$



$$\begin{aligned}
 \text{Argus depth} - \text{Hercules depth} &= \text{delta depth} \\
 \text{Argus depth} - 1238 &= 18 \\
 \text{Argus depth} &= 1220 \text{ meters}
 \end{aligned}$$



Section Two - Search Area on the Seafloor

When *Argus* holds stationary, *Hercules* can explore any part of the seafloor within an area below. (shaded in grey, not to scale) The horizontal range is the radius of this circle. The following questions will ask you to solve for the total seafloor area (area of a circle) that *Hercules* can cover while the vehicles are in different configurations.

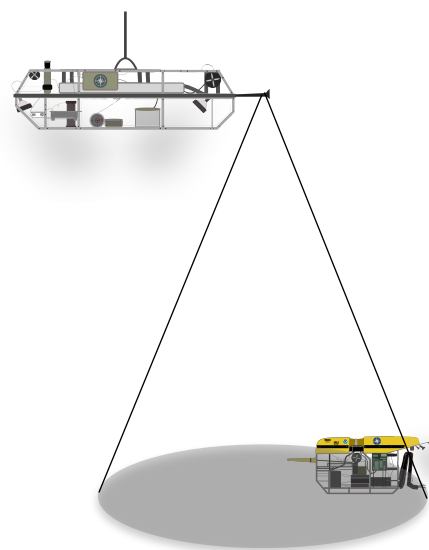
4. *Argus* and *Hercules* are connected by a 30 meter tether and the vehicles are flying with a delta depth of 22 meters. What is the seafloor search area available to *Hercules*?

$$\begin{aligned}
 a^2 + b^2 &= c^2 \\
 \text{horizontal range}^2 + \text{delta depth}^2 &= \text{tether}^2 \\
 \text{horizontal range}^2 + 22^2 &= 30^2 \\
 \text{horizontal range}^2 + 484 &= 900 \\
 \text{horizontal range}^2 &= 416 \\
 \text{horizontal range} &= \sqrt{416} \\
 \text{Horizontal range} &= 20.4 \text{ meters}
 \end{aligned}$$

Horizontal range is the radius of the search area

$$\begin{aligned}
 A &= \pi r^2 \\
 A &= \pi 20.4^2
 \end{aligned}$$

$$\text{Seafloor search area} = 1307 \text{ meters}^2$$





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5. If *Hercules* has a seafloor search area of 2653 square meters and a tether length 50 meters, what is the delta depth between *Hercules* and *Argus*?

$$A = \pi r^2$$

$$2653 = \pi r^2$$

$$\sqrt{(2653/\pi)} = r$$

$$\mathbf{r = 29.05 \text{ meters}}$$

Radius of the seafloor search area is the same as the horizontal range.

$$a^2 + b^2 = c^2$$

$$\text{horizontal range}^2 + \text{delta depth}^2 = \text{tether}^2$$

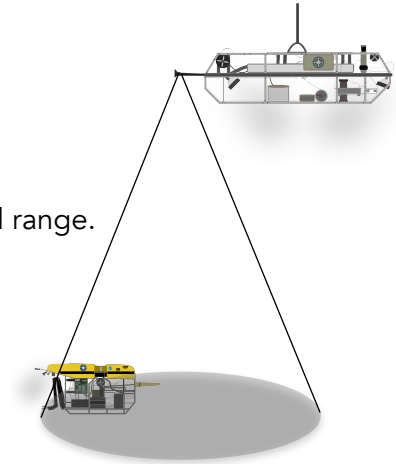
$$29.05^2 + \text{delta depth}^2 = 50^2$$

$$844 + \text{delta depth}^2 = 2500$$

$$\text{delta depth}^2 = 1656$$

$$\text{delta depth} = \sqrt{1656}$$

$$\mathbf{\text{delta depth} = 40.7 \text{ meters}}$$



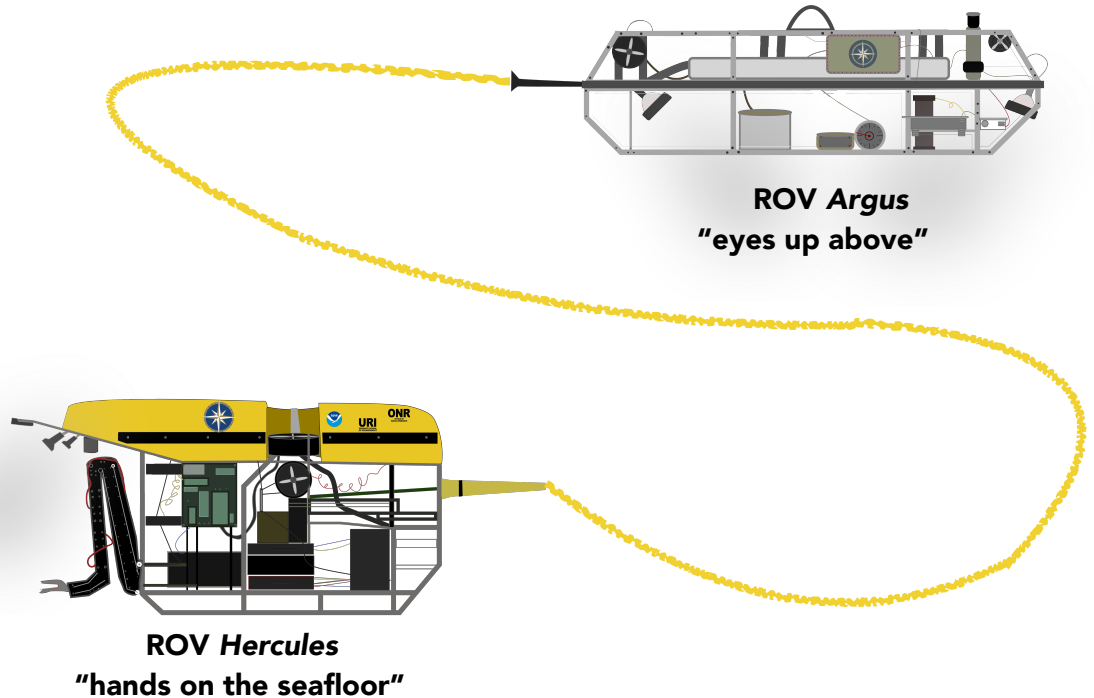


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Learning Goals

- Understand how to solve for an unknown dimension of a right triangle.
- Practice solving for the area of a circle using clues
- Understand the different distance relationships used by E/V *Nautilus'* ROVs while exploring the seafloor.

Introduction | Exploration Vessel *Nautilus* uses two remotely operated vehicles (ROVs) *Hercules* and *Argus* to investigate the biology, geology, and maritime history of unexplored parts of the seafloor.



These ROVs are constantly connected to the ship with an armored cable lowering the vehicles to the seafloor. ROV *Argus* hangs at the base of the stiff, metal cable beneath the back of the ship shining bright lights down onto the seafloor. ROV *Hercules* is connected to *Argus* using a flexible tether so it can move freely to explore the seafloor underneath the lights of *Argus*.

Hercules will always be somewhere beneath *Argus*. The difference between the depths of the two vehicles is expressed by the term **delta depth**. If you know the depth of the vehicles, the distance *Hercules* can move away from *Argus* can be easily calculated using trigonometry. Complete the worksheet below to explore the different configurations the ROVs can get into while diving.



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Helpful Resources:

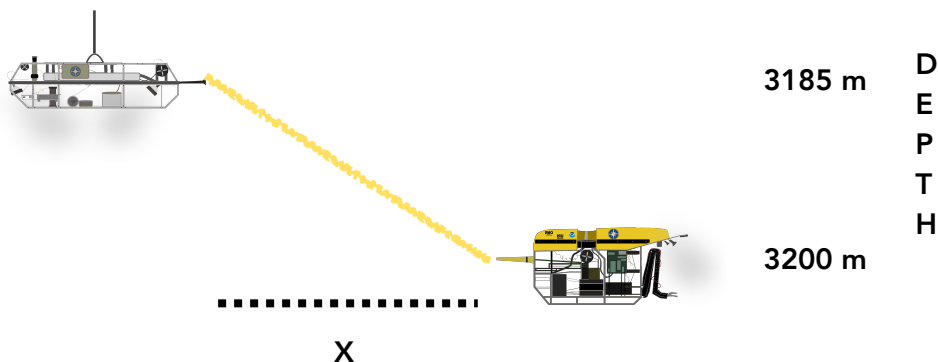
Ready to learn more about the ROVs Hercules and Argus?

Watch this video that explains the coordinated exploration technique that helps E/V *Nautilus* and her Corps of Exploration visit remote and poorly understood parts of our planet!
<http://nautl.us/1iUhPyu>

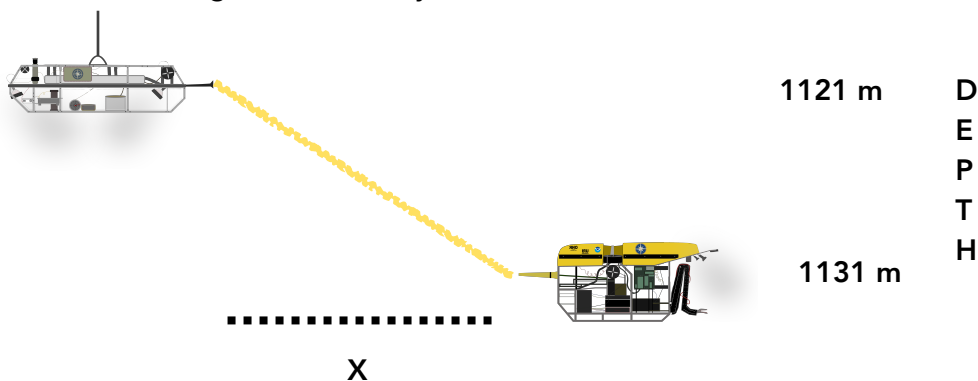
Behind the Science interview with ROV Pilot Todd Gregory about the challenges of using technology in the deep sea:
nautl.us/1VfIKWx.

Section One - Tether Management

1. If *Argus* and *Hercules* are connected with a 30 meter tether, solve for the horizontal range indicated by x .



2. If *Argus* and *Hercules* are connected with a 50 meter tether, solve for the horizontal range indicated by x .



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E/V *Nautilus* carries more than 4000 meters of cable to lower the ROVs to the seafloor. The cable is .68 inches in diameter and has braided steel armor on the outside to protect the delicate fibers in the inner core. The tether has two long glass strands called fiber-optics within the cable to conduct electronic commands from the pilots down to the ROVs and also to bring the high-definition video signal back to the surface. Within the cable are also three copper power supply lines that conduct high voltage electricity from the ship's generators down to the vehicles. The cable is wrapped carefully on a mechanical winch to raise and lower *Argus* during dives. Take a look at the cable below to see the delicate electronics that power *Hercules* and *Argus*.

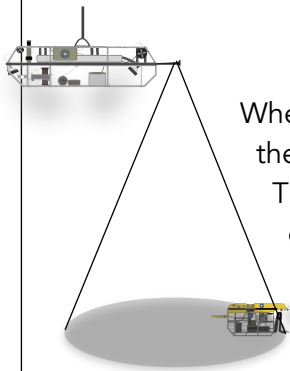


3. *Argus* and *Hercules* are connected with a 30 meter tether and the horizontal range is 24 meters across the seafloor. What depth is *Argus* sitting at?

1238 meters



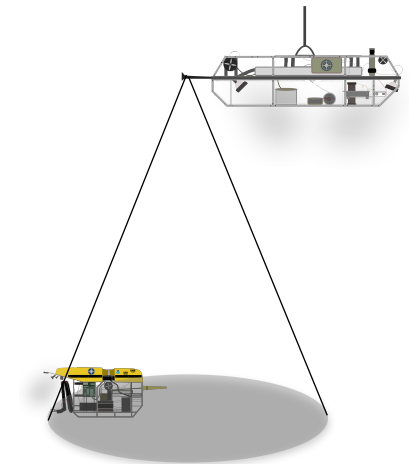
24 meters



Section Two - Search Area on the Seafloor

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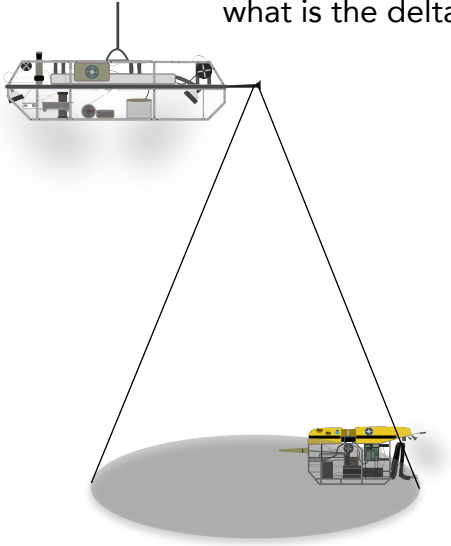
4. *Argus* and *Hercules* are connected by a 30 meter tether and the vehicles are flying with a delta depth of 22 meters. What is the seafloor search area available to *Hercules*?





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5. If *Hercules* has a seafloor search area of 2653 square meters and a tether length 50 meters, what is the delta depth between *Hercules* and *Argus*?



HOW LARGE IS NAUTILUS NATION?

Tracking the reach of Ocean Exploration Trust's education programs is essential in ensuring we are funded to continue making discoveries and inspiring the next generation of explorers.

Name: _____ **My Community (City, State):** _____

Email Address: _____

School's Name: _____

Instruction date: _____ **Grade level instructed:** _____

Subject area: _____

My education space is a...	Who did you engage in your teaching?
<input type="checkbox"/> Classroom	# Students
<input type="checkbox"/> After school program / Club meeting	
<input type="checkbox"/> Fair / Festival / Event	
<input type="checkbox"/> Museum / Science Center	# Community Members
<input type="checkbox"/> Other. Tell us more: _____	

Select all the OET materials you used in your instruction:

- ☐ STEM Learning Modules. Which ones? _____
- ☐ Digital Resource Library materials. Which ones? _____
- ☐ Nautilus Live website: photo albums ☐ highlight videos ☐ live stream
- ☐ Meet the Team STEM mentor profiles
- ☐ Facebook (NautilusLive) ☐ Twitter (@EVNautilus) ☐ Instagram (@nautiluslive)
- ☐ Other. Tell us more: _____

What made working with OET resources valuable to your instruction (select all that apply)?

- ☐ Hands-on activities ☐ STEM career connections
- ☐ Easy to use lessons ☐ Standards-based lessons
- ☐ Website resource access ☐ Real world application of curricula topics
- ☐ Excitement of cutting-edge discoveries / Unfamiliarity of deep ocean
- ☐ Another reason. Tell us more: _____

Using OET resources increased my confidence in teaching my science, technology, engineering, or math subjects.	<input type="checkbox"/> Yes	<input type="checkbox"/> No
OET provided me with helpful and relevant teaching resources.	<input type="checkbox"/> Yes	<input type="checkbox"/> No
Using OET resources increased my awareness of STEM careers.	<input type="checkbox"/> Yes	<input type="checkbox"/> No
If yes, how so? How can we improve?		

Please scan this document or snap a picture of it with your phone. Email the feedback or questions to education@oet.org. You can also submit feedback online: <http://nautl.us/2cp3PNu>

THANK YOU FOR ALL YOU DO!